Numerical studies of relativistic jets

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Numerical Studies of Relativistic Jets

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Abstract.
The connection between collimation and acceleration of magnetized relativistic jets is discussed. The focus is on recent numerical simulations which shed light on some longstanding problems.

1 Introduction

Understanding the physics of relativistic jets, notably their acceleration and collimation, has been a major area of study for several decades. It is widely agreed that jets are a magnetic phenomenon. The basic paradigm involves a bundle of field lines that is attached at its base to a central compact star – black hole (BH) or neutron star (NS) – or to an accretion disk. Rotation of the star and/or disk causes a helical outgoing magnetohydrodynamic (MHD) wave which accelerates any gas that is frozen into the field lines.

A great deal of analytical work has been done on jets, and the reader is referred to Begelman & Li (1994), Vlahakis & Königl (2003a,b), Beskin & Nokhrina (2006), Narayan et al. (2007), Lyubarsky (2009a,b), and references therein, for an introduction to the vast literature. However, the equations describing MHD jets are nonlinear and the problem is complicated, so there is a limit to what can be accomplished purely analytically. Fortunately, in recent years, numerical investigations have begun to contribute to the field, facilitated by the development of robust relativistic MHD codes (Koide et al. 2000; Komissarov 2001; McKinney & Gammie 2004; De Villiers et al. 2005; McKinney & Narayan 2007a,b; Komissarov et al. 2007, 2009; Tchekhovskoy et al. 2008, 2009a,b,c).

2 Force-Free Jets

Before considering the full MHD problem, it is useful to focus first on force-free jets. The force-free approximation is a simplification of ideal MHD in which we include electric and magnetic fields, and the corresponding charges and currents, but we ignore the inertia of the gas. The problem thus reduces to pure electrodynamics in a perfectly conducting medium. This very simple approximation provides surprisingly useful insight into the general problem.

2.1 Collimation

Michel (1973) derived an analytical solution for a force-free wind from a rotating star. He assumed a split monopole configuration in which the magnetic field is radial and has a constant strength at the surface of the star, with lines pointed into the star over one hemisphere and out over the other hemisphere.

The rotation of the star causes the magnetic field to develop a toroidal component B_ϕ which dominates with increasing distance: $B_\phi = -(\Omega R/c)B_p$, where Ω is the angular velocity of the star, R is cylindrical radius with respect to the rotation axis, and B_p is the field strength in the poloidal ($r\theta$) plane. However, even though B_ϕ dominates over B_p at large radii ($-B_\phi/B_p = \Omega R/c \gg 1$), nevertheless there is no change in the poloidal geometry – the poloidal field continues to remain radial. This is surprising since the tension associated with toroidal field curvature, the so-called hoop stress, is very large. Why does this not collimate the jet? It turns out that hoop stress is pefectly canceled by an electric force in the opposite direction, and hence there is no net tendency to collimate the flow. (This is a purely relativistic phenomenon – the electric field is negligible in nonrelativistic MHD.)

A similar result is seen in the paraboloidal force-free solution derived by Blandford (1976) and Blandford & Znajek (1977), and a more general class of self-similar force-free solutions studied by Narayan et al. (2007). Although hoop stress is very large at large distances from the base of the jet, once again it has small or no effect on the collimation of the jet.

All this suggests that a relativistic jet is unlikely to self-collimate. It needs to be collimated by an external agency. Assuming that the jets in active galactic nuclei (AGN) and X-ray binaries (XRBs) are associated with field lines from a spinning accreting BH, these jets must be confined by a wind from the surrounding accretion disk. In the collapsar model of gamma-ray bursts (GRBs), the confinement is presumably due to the stellar envelope of the collapsing star. If no confining medium is present, as in the case of relativistic outflows from radio pulsars, the flow is nearly radial.

2.2 Acceleration

Force-free jets accelerate very efficiently. In the simplest geometries, viz., split monopole and paraboloidal, the Lorentz factor of the outflow varies asymptotically as $\gamma \approx \Omega R/c$, increasing linearly with cylindrical radius. In the case of a paraboloidal jet, $\gamma \propto z^{1/2}$ (since $R \propto z^{1/2}$), where z is distance along the jet.

For more general geometries such as $R \propto z^{(2-\nu)/2}$ (Narayan et al. 2007), γ depends both on the value of $\Omega R/c$ and on the poloidal curvature of field lines. As a result, the run of γ with z exhibits two distinct asymptotic regimes (see Tchekhovskoy et al. 2008 for details). Nevertheless, at any given z, the field line with the largest Lorentz factor still satisfies $\gamma \propto z^{1/2}$. Thus, this simple and convenient scaling appears to be fairly general.

2.3 Jet Power

Since a force-free jet is purely electromagnetic, the energy flux is given by the Poynting vector $\vec{E} \times \vec{B}/4\pi$ (we set $c = 1$). For a rotating axisymmetric jet, $E = (\Omega R/c)B_p$ and is pointed perpendicular to the poloidal field line in the

$(r\theta)$ plane. In a relativistic jet electric and magnetic fields are nearly equal in magnitude (§2.1), $E \approx |B_\phi|$, therefore the poloidal component of the energy flux is given by $E|B_\phi|/4\pi = (\Omega R/c)^2 B_p^2/4\pi$. Applying this result at the BH horizon, the total electromagnetic power flowing out of a spinning BH is given by $P_{\rm jet} = k\Phi_{\rm tot}^2\Omega_H^2$, where $\Phi_{\rm tot}$ is the total magnetic flux threading the horizon, Ω_H is the angular frequency of the spacetime at the horizon, and k is a constant which depends on the field geometry (see Tchekhovskoy et al. 2009b for details). Although this result for $P_{\rm jet}$ is derived for a force-free jet in the limit of a slowly spinning BH (Blandford & Znajek 1977), it turns out to be surprisingly accurate even for rapidly spinning BHs, and also for MHD jets (Tchekhovskoy et al. 2009b).

One interesting question is whether the Ω_H^2 scaling of power is sufficient to explain the radio loud/quiet dichotomy of AGN. Observations indicate a factor of 1000 difference in radio power between radio loud and quiet AGN (Sikora et al. 2007). Can this be entirely due to differences in the BH spins of the underlying populations? Tchekhovskoy et al. (2009b) show that a model in which the BH is surrounded by a thick accretion disk, such that the jet subtends only a narrow solid angle around the axis, has a steep variation of jet power with spin – $P_{\rm jet} \propto \Omega_H^4$ or even Ω_H^6 (a similar steep dependence was observed by McKinney 2005 in the simulations of BHs with thick turbulent tori). Such a model is perfectly compatible with the observed radio loud/quiet dichotomy, whereas a BH with a thin disk around it would have $P_{\rm jet} \propto \Omega_H^2$ and is not likely to have a large range of jet power. This explanation of the radio loud/quiet dichotomy requires AGN jets, especially in low-luminosity systems, to be associated with thick advection-dominated accretion flows (ADAFs). There is, in fact, independent evidence for such a connection (Narayan & McClintock 2008).

2.4 Stability

According to the Kruskal-Shafronov criterion (e.g., Bateman 1978), cylindrical MHD configurations in which the toroidal field dominates are violently unstable to the $m = 1$ kink (or screw) instability. Since all models of relativistic magnetized jets have $B_\phi \gg B_p$, jets ought to be highly unstable. This has been recently confirmed by numerical simulations (Mizuno et al. 2009). Why are the jets observed in Nature coherent over enormous length scales?

Narayan, Li, & Tchekhovskoy (2009) studied the stability of force-free jets and found that the growth rate of the kink mode is quite low. There are two reasons for this. First, although jets may have $B_\phi \gg B_p$ in the "lab frame," they typically have $B_\phi \sim B_p$ in the comoving frame of the fluid. This makes the instability less severe. Second, as a result of time dilation, the growth rate as measured in the lab frame is further suppressed by a factor of $1/\gamma$. Note that the above study was limited to a very simple force-free model. The stability of more realistic relativistic MHD jets is an open question.

3 Relativistic MHD Jets

We now consider MHD jets with gas inertia, though, for simplicity, we ignore gas pressure. A steady axisymmetric MHD flow has a number of conserved

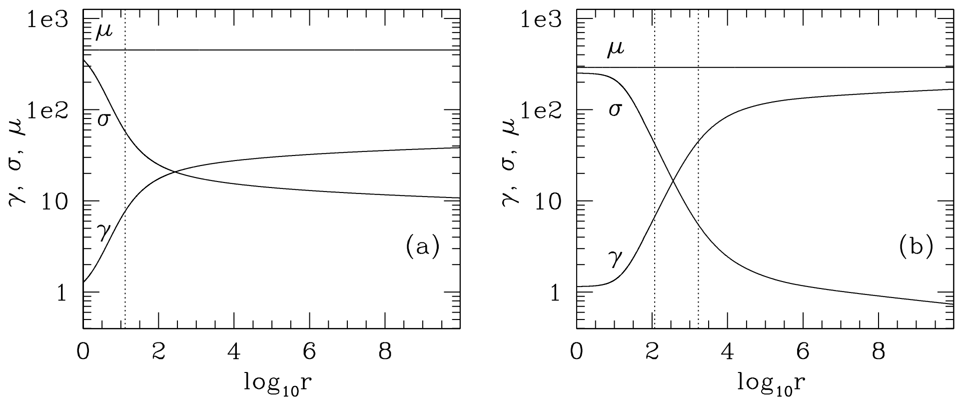

Figure 1. [Panel (a)] Solid lines show Lorentz factor γ, magnetization parameter σ and conserved energy flux μ (= 450) as a function of distance r (roughly in units of the BH horizon radius) along an equatorial field line ($\theta = 90°$) for a mass-loaded split-monopole outflow. Initially, γ increases rapidly with r. However, beyond the fast point (indicated by the vertical dotted line), it grows only logarithmically. As a result, even at astrophysically relevant distances, γ is only ~ 40 and most of the energy remains locked up in the magnetic field: $\sigma \sim 10$. This is the σ problem. [Panel (b)] Similar to panel (a), but for a polar field line at $\theta = 10°$. The two vertical dotted lines show the positions of the fast point (left) and the causality point (right). Here the flow continues to accelerate beyond the fast point and up to the causality point. As a result, it is less affected by the σ problem and achieves a larger final Lorentz factor $\gamma_{\rm final} \sim \mu$. (Based on Tchekhovskoy et al. 2009a)

quantities: (i) The angular velocity Ω is constant along any field line. (ii) The net mass flowing along a bundle of field lines is conserved, which means that the quantity $\gamma\rho v_p/B_p$ is constant along each field line, where ρ is the mass density in the rest frame of the gas and v_p is the poloidal 3-velocity of the gas in the lab frame. (iii) The angular momentum flowing along the bundle of lines is conserved. (iv) Finally, the energy flowing along the bundle is also conserved, which implies that

$$\mu = \left\{|\vec{E} \times \vec{B}_\phi/4\pi| + (\gamma^2\rho v_p)\right\}/\gamma\rho v_p = \text{ constant}, \tag{1}$$

where the first term in the numerator is the Poynting energy flux and the second is the gas kinetic energy flux. This equation may be rewritten as

$$\mu = \gamma(\sigma + 1), \qquad \sigma = |\vec{E} \times \vec{B}_\phi/4\pi|/(\gamma^2\rho v_p), \tag{2}$$

where the magnetization parameter σ represents the ratio of electromagnetic to kinetic energy flux at any point in the flow.

3.1 Acceleration and the σ Problem

Equation (2) has a simple interpretation. At the base of the jet, before the gas has accelerated, we have $\gamma \approx 1$, and so the energy flow is almost entirely in the form of Poynting flux: $\sigma \gg 1$, $\mu \gg 1$. Here the jet is magnetically very dominated and it behaves for all intents like a force-free jet. As the jet moves out, γ increases and the kinetic energy flux grows at the expense of the Poynting flux. Thus σ decreases as γ increases, but in such a way as to keep μ constant. If the acceleration is efficient, then we expect σ to become vanishingly small at large distance and all the initial energy to end up in the gas; we would then have $\gamma = \gamma_{\rm max} = \mu$. In fact, this ideal is rarely achieved.

The problem is most clearly seen in the split monopole geometry, where analytical results are available for equatorial field lines (Michel 1969; Beskin et al. 1998; Tchekhovskoy et al. 2009a). The flow accelerates initially just as in the force-free case, with $\gamma \sim \Omega R/c$. However, once the flow speed exceeds the fast magnetosonic wave speed in the medium, which happens when $\gamma \sim \mu^{1/3}$, or equivalently when $\gamma \sim \sigma^{1/2}$ (from eq. 2 with $\mu \gg 1$), the gas loses contact with gas behind it and acceleration slows down drastically. Beyond this point, there is only a growth in γ of order a logarithmic factor, so the final γ is given by $\gamma_{\rm final} \sim C\sigma^{1/2}$, where C is a logarithm. If μ is large, as it must be for a highly relativistic flow, the asymptotic value of σ remains large and most of the energy is still carried in the form of Poynting flux rather than as gas kinetic energy. This is the σ problem.

The σ problem is illustrated by a numerical example in Fig. 1a. As we see, the problem is not a lack of energy. In this example, $\mu = 450$ which means there is plenty of energy available and the gas could in principle accelerate up to $\gamma_{\rm max} \sim 450$. However, it only accelerates up to $\gamma_{\rm final} \simeq 40$ because the energy remains stuck in the magnetic field.

Simulations such as those shown in Figs. 1–2 of this paper are challenging and have only recently become possible with the use of the general relativistic MHD code HARM (Gammie et al. 2003), including recent improvements (Tchekhovskoy et al. 2007, 2008). There are two difficulties with these calculations. First, the large value of σ means that the electromagnetic and kinetic terms in the energy equation are very different in magnitude. The equations are thus stiff, making it very difficult to maintain accuracy. Second, magnetic acceleration is relatively slow: γ varies only as $z^{1/2}$ even when acceleration is efficient, and it grows logarithmically when it is inefficient. Therefore, in order to obtain useful results, it is necessary to simulate relativistic jets over very large length scales, which is obviously challenging. As of this writing, only one other group (Komissarov and collaborators) is able to carry out such calculations.

3.2 No σ Problem for Collimated Jets!

Observations suggest that relativistic jets in Nature do not suffer from the σ problem. Lind et al. (1989) showed that the interaction of a jet with an external medium is very different depending on whether $\sigma \ll 1$ or $\sigma \gg 1$. In the former case there is a clear hot spot (or working surface) where the jet shocks with the medium, and there is a well-defined backflow in a cocoon. This is similar to what is observed in FR II jets. In contrast, a magnetically dominated jet drills through the medium and has hardly any cocoon. It appears that AGN jets manage to achieve small values of σ before they shock on the external medium. Why are they not limited by the σ problem?

A key clue was provided by Tchekhovskoy et al. (2009a) who showed that equatorial and polar field lines in the split monopole geometry behave very dif-

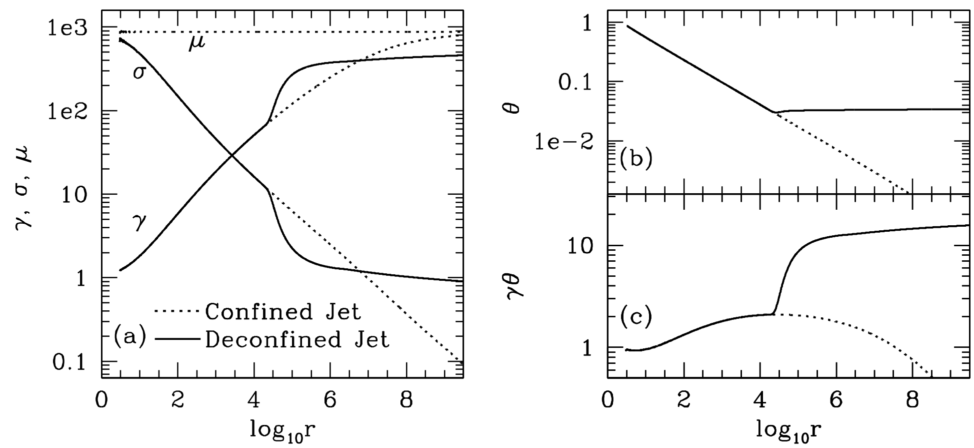

Figure 2. The dotted lines in panel (a) show the variation of γ, σ and μ ($\approx$ 900) as a function of distance r (BH units) for an MHD jet that is confined by a rigid wall with a shape given by $R \propto z^{5/8}$. As for all externally collimated jets, we see that this model does not suffer from the σ problem. Asymptotically, the jet achieves a large Lorentz factor $\gamma \approx \mu$ and a low magnetization $\sigma \ll 1$. The dotted line in panel (b) shows the variation of the jet opening angle θ, and in panel (c) the product $\gamma\theta$. Note that $\gamma\theta \lesssim 1$, which is a general feature of fully collimated jets. Solid lines in the three panels correspond to a jet that is confined out to a distance $r = 10^{4.5}$ and is then allowed to move freely in vacuum. The jet undergoes a period of rapid acceleration just as it exits into vacuum (see panel a), but its opening angle does not change (panel b). As a result, asymptotically, the jet has nearly the same Lorentz factor as in the previous example, $\gamma \sim \mu/2$, $\sigma \sim 1$, but now $\gamma\theta \sim 15$ (panel c). This model may be relevant for understanding GRB jets. (Based on Tchekhovskoy et al. 2009c)

ferently. This is illustrated in Fig. 1b which shows that acceleration along a polar field line continues well past the fast magnetosonic point. In fact, acceleration slows down only after the gas crosses another critical point, the "causality point," beyond which it can no longer communicate with the axis. The introduction of this new critical point means that the final Lorentz factor of the gas is a function of the angle θ between the poloidal field and the axis:

$$\gamma_{\rm final} \sin\theta \sim C\sigma^{1/2}, \tag{3}$$

where C is the same logarithmic factor as before. The smaller the value of θ, the larger the $\gamma_{\rm final}$ that the gas can achieve for a given value of μ. Thus, flows that remain close to the axis accelerate more efficiently.

This result is confirmed by numerical simulations of paraboloidal and other types of confined jets. The dotted lines in Fig. 2 show an example of a confined jet with $\mu \sim 10^3$. This jet accelerates with no trouble up to $\gamma_{\rm final}$ of nearly 10^3, achieving an asymptotic $\sigma < 0.1$. The lesson from this example is that, so long as a jet is provided adequate collimation, it will accelerate smoothly without being limited by the σ problem.

3.3 How are γ and θ related?

One additional interesting result is seen in Fig. 2: while γ increases, the jet angle θ simultaneously decreases. In fact, it appears that $\gamma\theta \sim 1$, as noted by Komissarov et al. (2009). Do relativistic jets always satisfy this constraint? This can be tested with GRB jets, where observations provide strong constraints on both γ and θ (e.g., Piran 2005).

Long duration GRBs typically accelerate to $\gamma \sim$ few hundred before decelerating in their afterglow phase. At the same time, jet breaks observed in GRB afterglows indicate typical jet opening angles $\theta \sim 0.05$ radian. Thus, GRB jets apparently have $\gamma\theta \sim$ few tens. This appears to violate the Komissarov et al. (2009) relation mentioned above, which is also confirmed by Fig. 2.

Fortunately, there is a simple solution. The solid lines in Fig. 2 show a second simulation in which the jet is initially confined over a few decades in distance and it is then allowed to move freely in vacuum. This is meant to mimic a collapsar in which the jet is confined so long as it is inside the stellar envelope, but becomes free once it escapes from the star. In this numerical example, we see that the gas experiences a burst of acceleration just as it is deconfined, without changing its opening angle very much. Asymptotically, the jet satisfies the scaling given in equation (3) with $C \sim 10$. The particular simulation shown here has $\gamma_{\rm final} \sim 500$, $\theta_{\rm final} \sim 0.04$, $\sigma_{\rm final} \sim 1$, which is close to typical values observed in GRBs. (Note that $\sigma_{\rm final}$ is not directly measured. However, the fact that GRBs emit a substantial fraction of their enerrgy in prompt emission implies that their jets cannot be Poynting-dominated.)

There is room for further work in this area. For instance, if observations by the Fermi Observatory routinely find $\gamma_{\rm final} > 10^3$ (Kumar & Barniol Duran 2009) and $\theta_{\rm final} > 0.1$ (Cenko et al. 2009) for many GRBs, we would need to explain why these jets have $\gamma\theta > 100$. The constant C in equation (3) is a logarithmic factor and is unlikely to be much larger than about $10-20$. Therefore, either we must accept that $\sigma \gg 1$, i.e., GRB jets are Poynting-dominated and somehow manage to convert a large fraction of their energy to promt gamma-rays, or that MHD is not the appropriate framework for understanding GRB jets.

4 Conclusion

The main message of the work described here is that collimation is the key to relativistic jets. If a jet is suitably confined by an external medium, it will accelerate without difficulty. While GRB jets have a natural collimating medium in the surrounding stellar envelope, the situation is less obvious for jets in AGN and XRBs. A disk wind is the most likely collimator in these systems, which suggests that many prominent jets are probably associated with geometrically thick accretion disks with strong winds and outflows, i.e., ADAFs of various kinds (Narayan & McClintock 2008). This connection is worth exploring further.

Equation (2) shows that the maximum Lorentz factor that a jet can achieve is equal to μ. A highly relativistic jet requires a large value of μ, which means that the jet must start out very magnetically dominated at its base with negligible mass-loading. There is very little understanding of how mass-loading works. Qualitatively, one imagines that field lines that are connected to the disk have a ready supply of disk gas for mass-loading, whereas lines that penetrate the

BH horizon are more likely to be mass-free. One thus suspects that the most extreme jets probably emerge from spinning BHs rather than from disks.

Finally, in all of our discussion we assumed that a coherent magnetic field is already present in the system. The origin of this field is not understood. It could be advected in by the accretion disk from outside. However, the efficiency of such advection is poorly understood and the topic is controversial.

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